Time-weighted volatility

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Abstract

We discuss the question of weekend and holiday market volatility in the options markets, its implications for the temporal interpolation of implied volatility, and the ramifications for numerical theta computations for the purpose of commercial P&L explanations. We give a practical methodology to accommodate these observations and requirements in a derivatives trading environment, and compare what this method implies for the future evolution of so-called "ON" options in the FX market with actual market-observed time series of such traded instruments' Black implied volatilities.

1 Introduction

The Black-Scholes-Merton [BS73; Bla76] forward option price formula ("BSM" for short), excluding any discounting from the valuation to the payment date, has the form

$$p = \phi \cdot [F \cdot \Phi(\phi \cdot d_1) - K \cdot \Phi(\phi \cdot d_2)] \tag{1.1}$$

with

$$\phi := \pm 1 \tag{1.2}$$

for calls and respectively puts, and

$$d_{1,2} \; := \; -\frac{\ln(K/F)}{\hat{\sigma}\sqrt{\tau}} \pm \frac{\hat{\sigma}\sqrt{\tau}}{2} \tag{1.3}$$

where F is the forward[1], K the strike, $\hat{\sigma}$ the so-called *Black implied volatility*, and τ the time to expiry computed from the calendar day count from the pricing date to the expiry date, divided by 365. Note that this *ACT365* rule to convert real world dates to the value for τ in the Black-Scholes-Merton (*Black*, for short) formula (1.1) has been in use for some considerable time, is enscribed in some regulatory (e.g., Basel) framework nomenclature, is often hard-coded in trading systems, and has been taught so in many if not all established business and banking courses at universities and other training institutions worldwide. It is essentially simply a convention that allows everyone to know precisely what they are dealing with when they are given an *implied volatility* $\hat{\sigma}$ without further specifications, which is of critical importance since the Black formula invokes the input volatility $\hat{\sigma}$ *always* directly in combination with $\sqrt{\tau}$. An inadvertent consequence to this convention is that each day is assigned the same amount of future daily variance of the underlying financial asset's future spot realisation.

Unfortunately, this homogeneous flow of time has the side effect that, for constant $\hat{\sigma}$ for options expiring before and after one specific weekend, the implication is that the respective option market maker is expecting the same amount of fluctuations to happen on the Saturday and Sunday as on the Friday before and the Monday after. This, alas, does not reflect the real world. A simple yet effective alternative is to use a different convention for the calculation of τ in the Black formula, the so-called *BUS252 volatility day count convention* where we only count active days of trading and divide them by (an estimate for the average) number of business days per year. Whilst this approach is reasonably well known among practioners, it rarely appears in the literature, one exception being [SV00]. For the respective trading desks, the BUS252 volatility day count convention is pragmatic and easy to use, though it has its drawbacks.

- First, as mentioned in [SV00], there is:

  > "*[...] the number of trading days is not necessarily proportional to the number of calendar days left over time, as there are more holidays during some seasons of the year. Therefore, one should consider using a different denominator during different times of the year, at least for pricing options with time to expiry less than a year.*"

- Second, the usage of a different volatility day count convention to the open market standard entails significant operational risk in the interaction with other departments, trading exchanges, market counterparties, and business IT systems.

- Third, in some markets such as FX, option market makers wish to assign to some business days *more* volatility than to any ordinary trading day because of expected announcements, e.g., monetary decisions, non-farm pay roll results, and so on.

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Key words: time-weighted options volatility, out of hours trading, weekend volatility, numerical theta, P&L explanation.

[1] i.e., the par strike of a cash-settled forward contract with the same expiry date

In this article, we aim at providing a practically usable approach to mark and interpolate implied volatility (in time) such that low-volatility periods such as weekends and holidays, as well as higher volatility days (or periods), are realistically accounted for, and yet we stick with the conventional *ACT365* interpretation of τ in the context of the Black formula. The core idea is very simple and has been around for a long time: the passing of calendar time into the future, in the context of options trading, represents the arrival rate of (mainly) microscopic market news, leading to (mainly) microscopic, i.e., diffusively random, underlying financial asset price movements. On weekends and holidays, this information rate is drastically reduced, amounting to a *slower flow of market time*, which in turn leads to less *variance* being attributed to such periods of time, and vice versa for days of anticipated extra influential announcements or similar. Whilst this is all very straightforward, we also discuss the ramifications for the standard *numerical theta* calculation, and how this can be amended in order to obtain a time-decay-of-value measure that is in line with trading practitioner's expectations for P&L explanation purposes. It is this latter part that is the main focus of this note.

2 Volatility time and Black-Scholes-Merton variance

Option quotes both on exchanges and the inter-dealer market via electronic market making data feeds and platforms such as Reuters and Bloomberg typically comprise sets of data for a discrete number of expiries, e.g., overnight, one week, one month, etc. These quotes may be for fixed expiry dates (i.e., the expiry date set does not change from one trading day to the next, with the exception perhaps of the front quote slipping into the past) as is usual for exchange-traded options, or given as tenors relative to the current date, e.g., "ON" representing always an option to the end of the next business day, "1W", "1M", etc. Either way, in-house, such quotes are usually converted into some sort of *implied volatility surface* representation, calibrated such that the input quotes are perfectly reproduced, and providing both temporal and strike-wise interpolation capabilities. In the following, we shall focus entirely on the temporal interpolation direction and ignore any strike dependence altogether for the sake of simplicity of presentation. This is not to say that the interpolation in the strike direction is of lesser significance. It merely is not the subject of this article.

Let us denote the set of input quote expiry dates as $\mathcal{T} := \{T_1, T_2, ..., T_n\}$, and the current valuation date as T_0. Associated with these expiry dates, we take as given a set of "implied" volatilities denoted as $\{\hat\sigma_1, \hat\sigma_2, ..., \hat\sigma_n\}$. We already alluded earlier to the fact that these volatilities can only be unambiguously converted into option prices if we have a clear definition as to the meaning of the time variable in the BSM formula (1.1). Hence, for this purpose, we consider as given also a *volatility time* function

$$\tau(T_a, T_b)$$

that converts the span between any two calendar dates T_a and T_b into a time quantity that can be used in the BSM formula. To put it differently, we state that any volatility $\hat\sigma(T)$ for expiry T corresponds to a T-forward option price out of the valuation date T_0 by the aid of the Black (or BSM-) *variance*

$$\hat v(T) \;:=\; \hat\sigma(T)^2 \cdot \tau(T_0, T) \tag{2.1}$$

via formula (1.1) with

$$d_{1,2} \;:=\; -\frac{\ln(K/F)}{\sqrt{\hat v}} \pm \frac{\sqrt{\hat v}}{2}\,. \tag{2.2}$$

3 Time-weighted volatility

Given a set of input quotes as described in the previous section, and obviously in ignorance of any strike dependency as we shall allow ourselves for the purpose of this article, the primary purpose of the volatility surface is to enable us to obtain option prices for any intermediate expiry dates not present in the input calibration set. This can in principle be done with *any monotonicity preserving* interpolation method on Black variance over expiries. For the purpose of our current context, however, we prefer to use the simple approach of linear interpolation in variance. In other words, for an input expiry date T that is not exactly one of our surface's calibration expiry set $\mathcal{T}$ (else we obviously just use the volatility for the quoted expiry), we find the bracketing expiries T_{i-1}, and T_i such that $T_{i-1} < T < T_i$ and set

$$\hat v(T) \;:=\; \hat v_{i-1} \cdot (1-w) + \hat v_i \cdot w \tag{3.1}$$

with

$$\hat v_i \;:=\; \hat\sigma_i^2 \cdot \tau(T_0, T_i) \qquad \forall \quad i = 1..n. \tag{3.2}$$

for a suitably computed interpolation coefficient w whose specifics will be discussed imminently. In the case that $T < T_1$, we set $i := 1$, and for $T > T_n$, we set $i := n$, but in both cases we use T_0 instead of T_{i-1} (and thus use 0 instead of $\hat v_{i-1}$), which renders the first term on the right hand side of (3.1) as identically zero. In order to be arbitrage-free and consistent at either end T_{i-1} and T_i, the interpolation coefficient $w = w(T)$ must rise monotonically from 0 at $T = T_{i-1}$ to 1 at $T = T_i$.

For the purpose of time-varying attribution of variance over all the calendar days from T_{i-1} to T_i, according to each day's trading significance, we allow for a *time-weighting* (or *time measure*) function

$$\omega(T_a, T_b).$$

A practical implementation of such a function is simply an iterative loop over all days in the interval summing up 1 for each ordinary business day, a chosen weight w_{W} assigned to weeekend days for each day on a weekend, a chosen weight w_{H} for each bank holiday (in practice we often use $w_{\text{W}} \equiv w_{\text{H}}$ as some very small but positive number such as 10^{-6}), and a number w_{E} larger than 1 for each day of anticipated extra trading activity (which addresses the third of the criticsms of the BUS252 day count convention in section 1), but more refined choices are of course equally possible where warranted by the respective trading environment. With this, we then use the simple formula

$$w(T) := \frac{\omega(T_{i-1}, T)}{\omega(T_{i-1}, T_i)} \tag{3.3}$$

and apply it in (3.1) to obtain *time-weighted* volatility interpolation.

Remark 3.1. Note that the formulation (3.3) obviates any kind of normalisation of the time-interval weighting function $\omega(\cdot,\cdot)$. Also note that the definition of $w(T)$ as a ratio of time-interval weights immediately addresses the first of the three criticisms of the simple BUS252 day count (volatility time) convention in the introduction as quoted from [SV00].

Remark 3.2. The conventional case of standard linear variance interpolation without explicit time-weighting is resumed in this setting by defining the *time-weighting* function $\omega(\cdot,\cdot)$ to be identical to the *volatility time* function $\tau(\cdot,\cdot)$, i.e., when

$$\omega(\cdot,\cdot) \equiv \tau(\cdot,\cdot) .$$

Note that this is not necessarily the same as the simple day weight summation rule for $\omega(T_a, T_a)$ with $w_{\text{W}} \equiv w_{\text{H}} \equiv w_{\text{E}} \equiv 1$ since the volatility time function $\tau(\cdot,\cdot)$ may have been specified originally as a BUS252 rule. However, in order to address the second of the three criticisms of the BUS252 convention in section 1, we obviously wish to use the standard ACT365 day count convention for $\tau(T_a, T_b)$ whenever we have expressly given time weight coefficients.

All in all, our *time-weighted* temporal interpolation rule for implied volatility is

$$\hat\sigma(T) = \sqrt{\hat\sigma_{i-1}^2 \cdot \frac{\tau(T_0,T_{i-1})}{\tau(T_0,T)} \frac{\omega(T,T_i)}{\omega(T_{i-1},T_i)} + \hat\sigma_i^2 \cdot \frac{\tau(T_0,T_i)}{\tau(T_0,T)} \frac{\omega(T_{i-1},T)}{\omega(T_{i-1},T_i)}} \tag{3.4}$$

for $T_1 < T < T_n$, where we have used the additivity relationship

$$\omega(T_{i-1}, T_i) = \omega(T_{i-1}, T) + \omega(T, T_i) \tag{3.5}$$

which hopefully holds for $\omega(\cdot,\cdot)$, and

$$\hat\sigma(T) = \hat\sigma_i \cdot \sqrt{\frac{\tau(T_0, T_i)}{\tau(T_0, T)} \cdot \frac{\omega(T_0, T)}{\omega(T_0, T_i)}} \tag{3.6}$$

for $T < T_1$ (i=1) or $T > T_n$ (i=n).

Remark 3.3. We see from (3.6) that without explicit time-weighting, i.e., when we set

$$\omega(\cdot,\cdot) \equiv \tau(\cdot,\cdot) ,$$

we simply obtain flat extrapolation at the front and at the back of our quote set, which is arguably simplistic yet perfectly pragmatic in a trading and hedging environment.

We show in figures 1 and 2 examples for the term structures of implied volatility over subsequent business days as expiries generated by the methodology discussed in this section. Note that the BUS252 "quotes" were generated by multiplying the corresponding ACT365 quotes by

$$\sqrt{\frac{\tau_{\text{ACT365}}(T_0, T_i)}{\tau_{\text{BUS252}}(T_0, T_i)}} .$$

Also, we should mention that in the absence of any special day add-on weights, and for holiday and weekend weights being zero, such that the time-weighting function $\omega(\cdot,\cdot)$ becomes the same as the business day count used in BUS252, the shown data for "BUS252" and for "ACT365 with time weights" in the two graphs correspond to identical option prices on *all dates*, not only on quoted expiries (which are always calibrated to exactly by all methods).

4 Theta

A common bone of contention between the various departments of any investment bank is the meaning, definition, and purpose of the so-called *theta*, i.e., the theoretically predicted time-decay of the value of any option. It is, at least philosophically, arguable whether such a quantity should be referred to in one and the same context as *market risk measures* since the progression of time is without doubt one of the few things that can be relied on with certainty, i.e., without any *risk* whatsoever. However, since the dynamic replication of any derivative's payoff is a balancing act between the net proceeds from the trading of the respective hedge instruments and the derivative's net-present-value *intrinsic* time-decay, it is clearly of interest

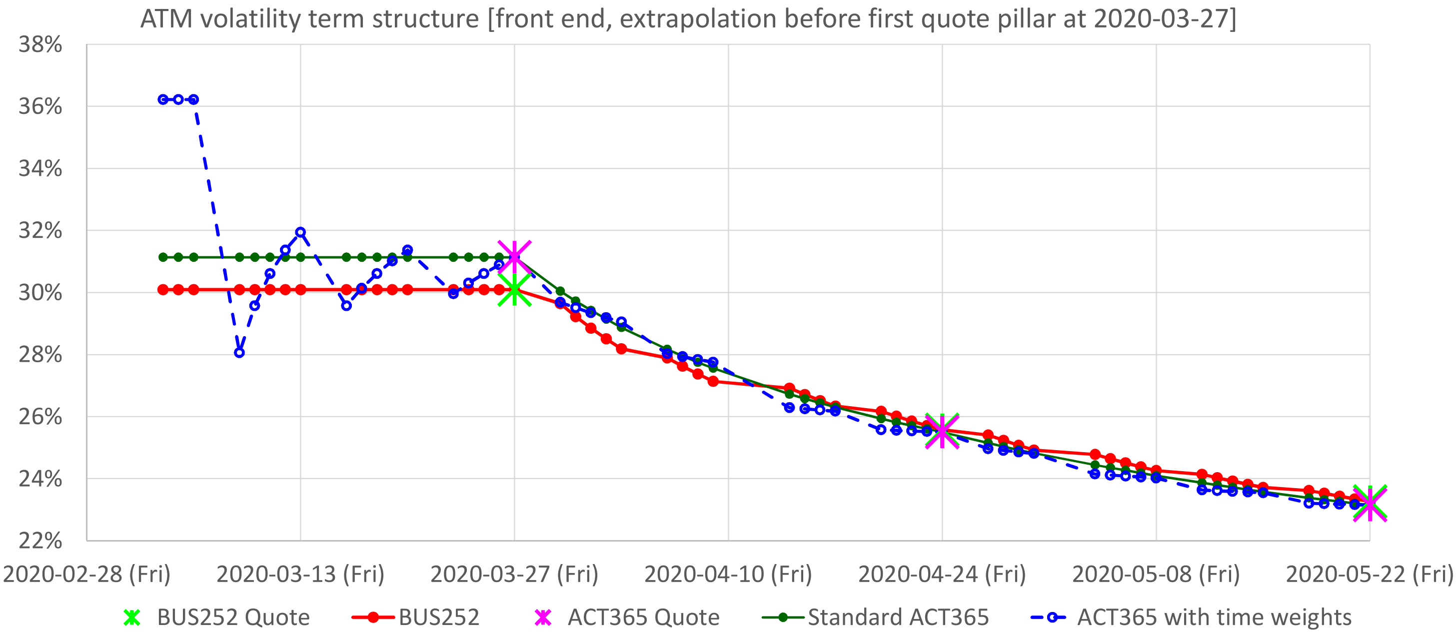

FIGURE 1: Implied volatilities as generated by inter- and extrapolation for short maturities.

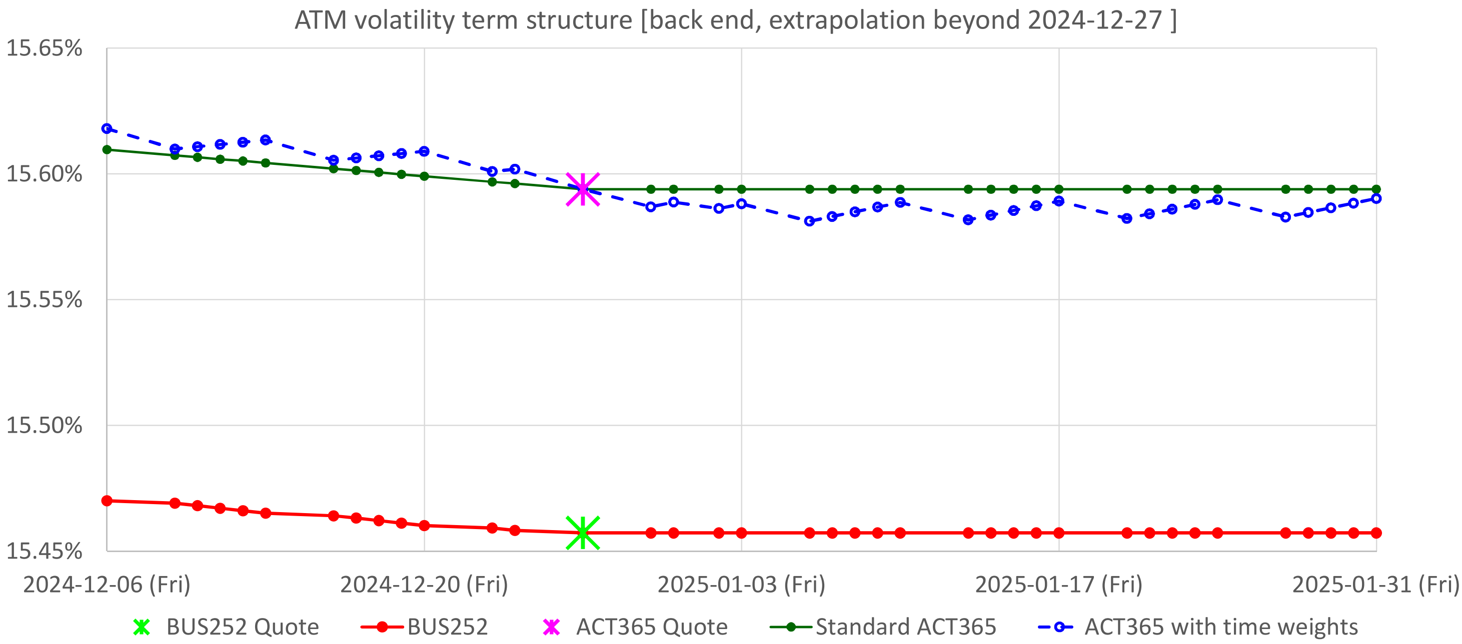

FIGURE 2: Implied volatilities as generated by inter- and extrapolation for long maturities.

to have a sensible measure of the amount of value decay that has to be made up by the frequent buying and selling of the hedges.

Probably the most widespread approach for a numerical (forward looking) *theta* computation of any vanilla option book is to take the entire market data set for any given valuation date T_0 and shift it to the following business day, here denoted as T_0', reprice all positions, and subtract the values of the same positions as already computed for T_0. For volatility surfaces, the standard practice seems to be to keep all volatilities for quoted expiries unchanged in the shift from T_0 to T_0', even when these expiries are given as fixed dates, as opposed to as periods, which we shall assume from here on. And this is where the simple BUS252 volatility day count convention shows its most popular feature that gave rise to it being favoured by many traders who wish to see if their day-to-day (note: business days!) delta-hedging of the option's gamma breaks even against the time decay (theta) of the position. In most financial markets, e.g., equity, FX, commodity, interest rates, historically, by far the most activity in price movements is observable during business days, and so the hedging practitioner prefers to see only one (business) day of time decay from a Friday to a Monday, just as they see from a regular Thursday to the subsequent Friday. Alas, when a volatility surface is configured simply with respect to the standard ACT365 volatility day count convention, and volatility quotes on the surface are kept constant when we shift $T_0 \to T_0'$ from a Friday to a Monday, any existing option position is revalued for T_0' with a time number going into the Black formula that has been reduced by a full *three* days. This leads to the so-computed numerical theta being about three times[2] as large as is likely to be made up by delta-hedging during the business hours of the Monday, which is only *one* business day. Whilst this may balance out over the course of the week, or multiple weeks, it is understandably a nuisance for the hedger, and hence traders prefer a different time decay measure.

Let us assume, at least initially, that the next business date T_0' succeding our original valuation date T_0 is still before the first quote pillar date T_1, i.e., $T_0 < T_0' < T_1$, and recall the definition (3.2) of the Black variances $\hat{v}_i$ associated with the surface's quote expiry dates. The purpose of the time weighting function $\omega(\cdot,\cdot)$ is to distribute the variance between T_0 and T_1 with respect to any intermediate date T^* (such that $T_0 < T^* < T_1$) according to:-

$$\begin{array}{ll} (1-w_1)\cdot\hat{v}_1 & \text{for the time span from } T_0 \text{ to } T^* \\ w_1\cdot\hat{v}_1 & \text{for the time span from } T^* \text{ to } T_1 \end{array} \tag{4.1}$$

with

$$w_1 \;:=\; \frac{\omega(T^*,T_1)}{\omega(T_0,T_1)}\,. \tag{4.2}$$

We now choose $T^* := T_0'$, and desire that the first quotation expiry (T_1) variance after the valuation date shift $T_0 \to T_0'$ should satisfy

$$\hat{v}_1' \;=\; w_1\cdot\hat{v}_1 \tag{4.3}$$

in accordance with the original idea of the variance distribution as governed by the time weighting function $\omega(\cdot,\cdot)$. This gives us

$$\hat{\sigma}_1'^{\,2}\cdot\tau(T_0',T_1) \;=\; \frac{\omega(T_0',T_1)}{\omega(T_0,T_1)}\cdot\hat{\sigma}_1^{\,2}\cdot\tau(T_0,T_1) \tag{4.4}$$

as the equation to define the volatility $\hat{\sigma}_1'$ for expiry T_1 on the 1-business-day-forward-propagated volatility surface. In total, we obtain

$$\hat{\sigma}_1' \;=\; \hat{\sigma}_1\cdot\sqrt{\frac{\omega(T_0',T_1)}{\omega(T_0,T_1)}\cdot\frac{\tau(T_0,T_1)}{\tau(T_0',T_1)}} \tag{4.5}$$

as the volatility quotation slice update rule required for the front slice (for expiry T_1) to be consistent with the concept of variance attribution according to the given time weighting function, and thus to give a profit-and-loss explanation (i.e., *theta* computation) in line with the respective holiday and weekend weighting.

Remark 4.1. Note that in the special case of $\omega(\cdot,\cdot) \equiv \tau(\cdot,\cdot)$, the square root term in (4.5) cancels out identically and we simply obtain $\hat{\sigma}_1' = \hat{\sigma}_1$, i.e., quote invariance under the valuation date shift $T_0 \to T_0'$.

In order to meet all of the following requirements:-

- in the absence of any time weighting, i.e., in the conventional standard case which is equivalent to the default choice $\omega(\cdot,\cdot) \equiv \tau(\cdot,\cdot)$, the volatility surface keeps all quotes invariant under the valuation date shift $T_0 \to T_0'$,
- the volatility surface behaves with respect to profit-and-loss explanation (i.e., *theta* computations) equally when
    a) it is configured without time weights but $\tau(\cdot,\cdot)$ is defined via BUS252 and
    b) it is configured with zero weekend and holiday weights and $\tau(\cdot,\cdot)$ via ACT365,

we ask that the translation rule

$$\hat{v}_i' \;=\; \frac{\omega(T_0',T_i)}{\omega(T_0,T_i)}\cdot\hat{v}_i \tag{4.6}$$

holds not only for $i=1$, but for all i. This means, when the volatility surface is shifted forward one (or multiple) business days, apart from that we drop all front quotation slices that thereby slip into the future past, we update all volatilities according to the *theta mutation logic*

$$\hat{\sigma}_i' \;=\; \hat{\sigma}_i\cdot\sqrt{\frac{\omega(T_0',T_i)}{\omega(T_0,T_i)}\cdot\frac{\tau(T_0,T_i)}{\tau(T_0',T_i)}}\,. \tag{4.7}$$

To demonstrate the efficacy of the above logic for volatility surfaces, we show in figure 3 the 1-business-day forward looking numerical theta of an ATM vanilla option during the last 30 business days of its life, as a function of business-days-left to expiry. The actual real-world time span comprised 44 calendar days. For the standard ACT365 volatility day count convention setting, we can clearly see the approximately 3 times larger magnitude of theta about once every 5 business days corresponding to Fridays, and one exceptionally large theta burst at t = 4-days-to-expiry of a factor somewhat larger than 5 which corresponds to the Thursday before Easter in 2020. This is because in the respectively applicable business calendar (TARGET) there is a 5 calendar day span from the last business day before Easter (Thursday) to the next after Easter (Tuesday). In contrast, we obtain

[2] Due to the fact that vanilla option prices are a concave function (proportional to the square root) of time to expiry, the numerical three-calendar-day theta will always be strictly more than a factor three of that of a one-calendar-day theta, with that factor even diverging as we approach the option's expiry.

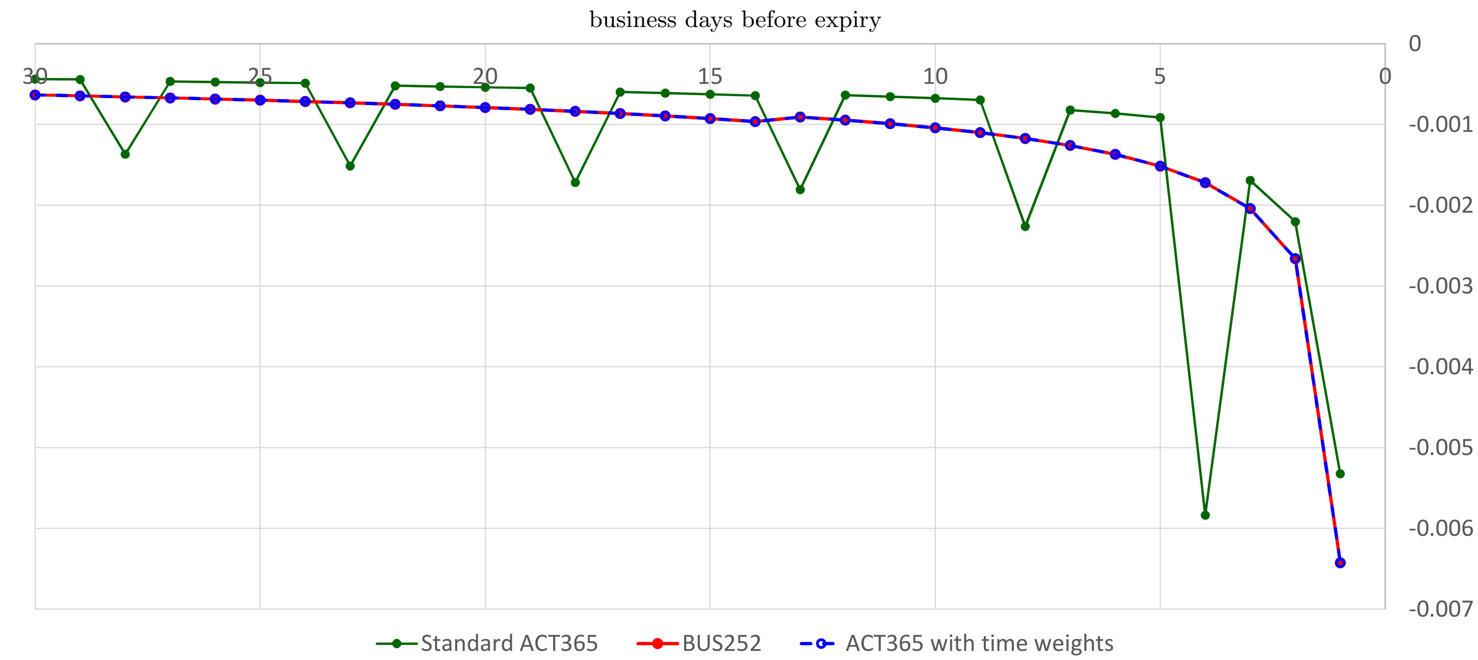

FIGURE 3: 1-business-day forward looking numerical theta of an ATM vanilla option.

identical output from the BUS252 method and from the ACT365 method with time weights configured to emulate simple business day counting (no special announcement day weights, weekends and holidays both effectively zero-weighted). We mention that the small kink at $t =$ 13-days-to-expiry for "BUS252" and "ACT365 with time weights" is due to a quote expiry on the surface slipping into the past. Since the chosen option's expiry date is not equal to any of the quoted expiries on the surface, this transition changes how the option's volatility is retrieved from the surface: up until 14 business days before expiry, it is interpolated over the two front quote expiries on the surface, but therafter, it is computed only from the front quote.

5 "ON" FX volatility time series

In the FX market, short-dated so-called *overnight* ("ON") options are traded anew on every business day. Each of these contracts starts trading in the morning of the business day in London, with expiry being the close of business the next day in New York. As usual, when prices are shown as volatilities, these are quoted in terms of the standard ACT365 time-to-expiry rule. Thus, even though they trade, in total, for two full business days (or arguably even a little bit more), on their first day of trading, the Black volatility is to be used with just one calendar day of time to expiry. It is also possible, however, that the *next business day* is one or several *calendar days* further away due to weekends or bank holidays, resulting in some interesting effects for the intra-day time series of these "ON" contracts' Black volatility. We show such a time series in figure 4 for EURUSD "ON" options from May 2020. Note that the seemingly continuous curve can be somewhat misleading to the uninitiated. First, pay attention to the fact that the enumeration along the abscissa only shows business days, and hence the curve even jumps through entire weekends. Second, be aware that the time series switches every morning to the next contract, which accounts for the sudden upwards jump at the beginning of each trading day. Further, we now discuss two very specific peculiarities that are a consequence of the ACT365 time convention in use with the Black formula.

1. Consider the relatively stationary volatility period of Tuesday the 12$^{\text{th}}$ to Wednesday the 13$^{\text{th}}$ of May in figure 4. On each of these mornings, the new option contract opened with about 9.7% of Black volatility. By the end of each of these business days, however, the implied Black volatility of the very same contract had gone down to about 6.8%. Why would that be? It turns out that the market's perceived *instantaneous* volatility had not changed at all! The FX options market, trading down to the minute as it does, trades these options as if they have a total life span of almost two real-world trading days. Thus, they open up with an associated Black variance (to the contract's expiry) that is about twice of what is left at the end of the first of its two trading sessions. Thus, to accommodate a decay of the Black variance $\hat{\sigma}^2 \cdot \tau$ (with $\tau = 1/365$ fixed throughout the whole first trading day!) of a factor of 2 from the start of the day when the options starts being quoted to the end of that day, the volatility must come down by about $1/\sqrt{2}$ since then $\left(\hat{\sigma}/\sqrt{2}\right)^2 \cdot \tau = \hat{\sigma}^2 \cdot \tau/2$. And lo and

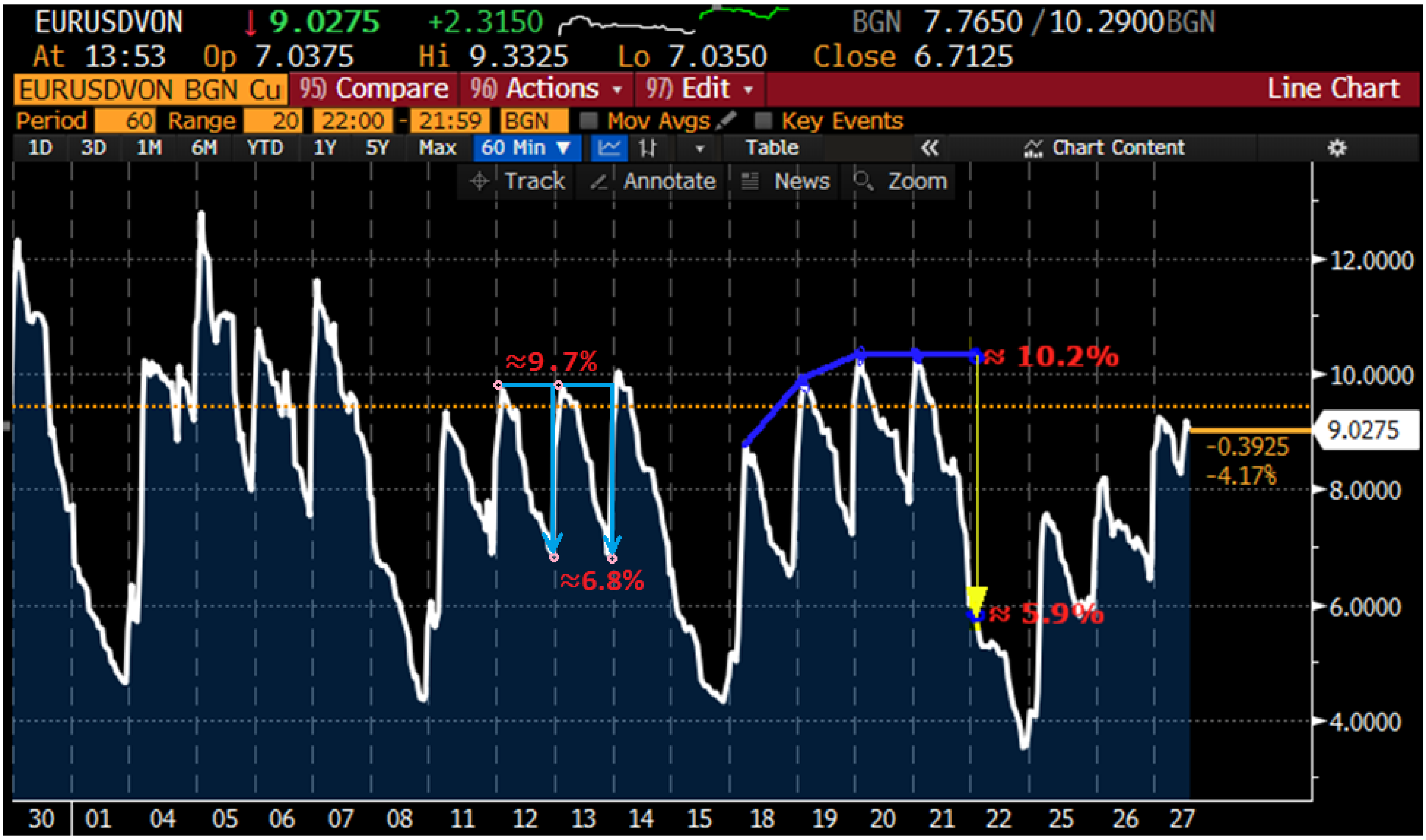

FIGURE 4: EURUSD "ON" option volatility time series Bloomberg screenshot from May 2020.

behold:

$$9.7\%/6.8\% \;\approx\; \sqrt{2}\;.$$

2. Even more intriguing is the weekend effect on Friday mornings: the new contract with 1B expiry now trades until Monday evening, and is accordingly quoted with a Black time-to-maturity of $\tau = 3/365$. The market, however, assigns little variability to the weekend, and prices it with approximately the same amount of market variance as if there was no weekend in the middle of the life of the Friday-to-Monday contract. Take the example of the Friday-to-Monday contract that started trading in the morning of 22nd of May in figure 4. Assume that the market assigned to it about the same Black variance at its start of trading as the preceding Thursday-to-Friday contract (which was about $\hat{\sigma} = 10.2\%$ with $\tau = 1/365$). Denote the Friday-to-Monday time-to-expiry as $\tau' = 3/365$ and its Black formula volatility as $\hat{\sigma}'$. Thus, we want to have

$$\hat{\sigma}^2 \cdot \tau \;\approx\; \hat{\sigma}'^2 \cdot \tau'\;.$$

We substitute $\tau = 1/365$ and $\tau' = 3/365$ and obtain

$$\hat{\sigma}' \;\approx\; \hat{\sigma}/\sqrt{3}$$

which agrees well with $5.9\% \approx 10.2\%/\sqrt{3}$!

Whilst we do not want to go into the fine details of the intra-day value decay of any one option contract, it is reassuring to know that the time-weighted implied volatility interpolation of section 3 realistically emulates the second of the above mentioned observations. This is shown in figure 5 where we used one volatility surface (originally for the 4$^{\text{th}}$ of March 2020, for a European stock with the TARGET business calendar) to generate a virtual "ON" (daily) volatility time series by shifting the surface to a whole sequence of forward dates (using the *theta mutation logic* presented in section 4, though that's less important here), and from each such forward surface read out the 1-business-day-expiry ATM volatility. We can clearly see in the "ACT365 with time weights" series the Thursday-to-Friday drop, and from the numbers we can confirm that it is by *exactly* a factor of $1/\sqrt{3}$ on all regular such days. There are two exceptions, the first being the 27$^{\text{th}}$ of March where we have the front quote expiry on the surface slipping into the past, and we incur a jump to the next front quote. The second exception is from Wednesday the 8$^{\text{th}}$ of April to the Thursday before Easter, where we can measure a drop of exactly a factor of $1/\sqrt{5}$, just as we would want it.

Happy days!

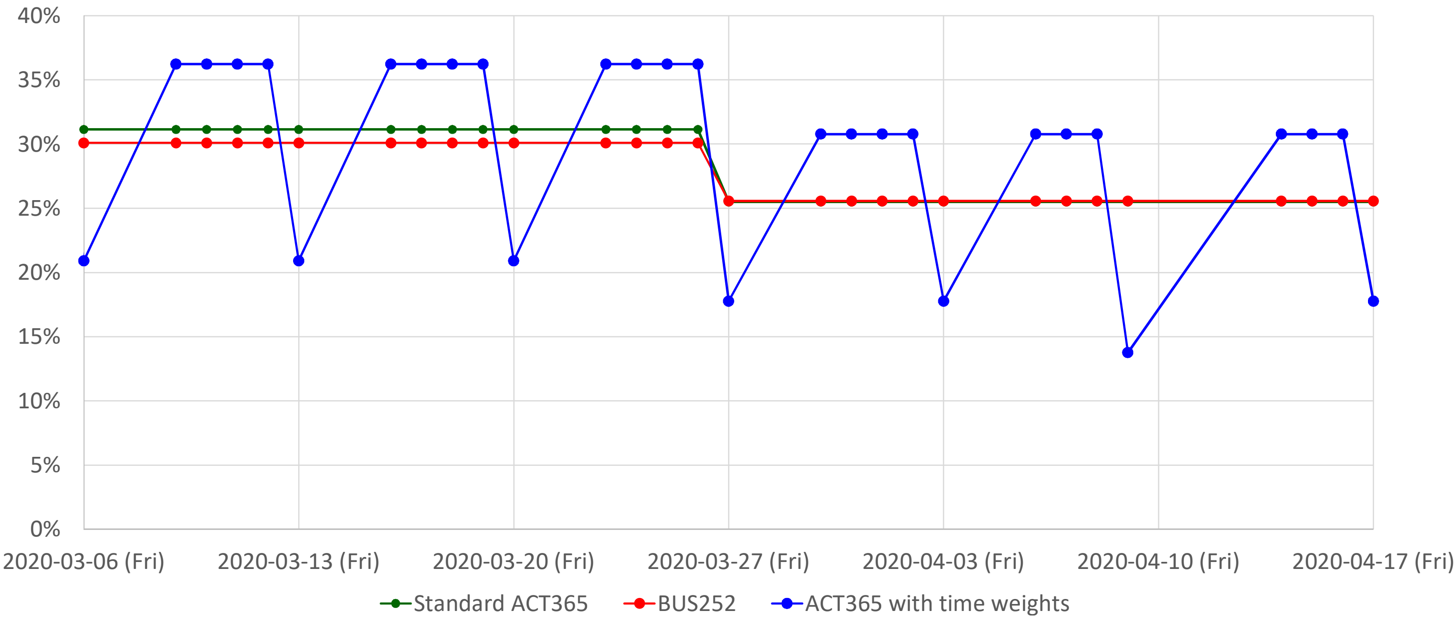

FIGURE 5: A virtual 1-business-day ("ON") volatility time series for subsequent calendar days. The "ACT365 with time weights" data set was generated by the time-weighted *theta mutation logic* of section 4 and the time-weighted volatility interpolation of section 3.

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