Corrections to "Monte Carlo methods in finance"

Below, I highlight the erroneous text in red, and the replacement in blue. **Please note that some of the corrections listed below may only apply to the very first print batch and may already be corrected in recent reprints.**

i. Chapter 2, section 2.5, equation (2.92), page 18 reads

$$\mathrm{V}\left[\xi_i\right] \;\; = \;\; \sigma$$

but should read

$$\mathrm{V}\left[\xi_i\right] \;\; = \;\; \sigma^2$$

The difference is that the variance should have been defined as the square of σ.

ii. Chapter 3, section 3.2, equation (3.5), page 24 reads

$$\mathrm{d}X_i = b_i(t, \boldsymbol{X})\,\mathrm{d}t + \sum_{i=1}^{n} a_{ik}(t, \boldsymbol{X})\,\mathrm{d}W_{k\,t}$$

but should read

$$\mathrm{d}X_i = b_i(t, \boldsymbol{X})\,\mathrm{d}t + \sum_{k=1}^{n} a_{ik}(t, \boldsymbol{X})\,\mathrm{d}W_{k\,t}$$

The difference is that the sum should be over $k = 1$ to n, *not* over $i = 1$ to n.

iii. Chapter 5, section 5.1, middle of page 43 reads

... the joint distribution density $\psi(x, y)$ is when both x and y are normal variates. In general, however, the inference ...

and should read

... the joint distribution density $\psi(x, y)$ in a natural (but not unique) way is when both x and y are normal variates: an obvious candidate for the joint distribution is then the bivariate normal distribution[*] In general, though, the inference ...

⋮

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[*] As explained later in section 5.2, this would correspond to the co-dependence structure being formed by the Gaussian copula. However, we could equally use any other copula to govern the co-dependence of the two marginally normally distributed variates x and y as long as the given correlation number is attainable by the co-dependence structure of two Gaussian densities under the chosen copula.

iv. Chapter 5, section 5.2, equation (5.17), page 45 reads

$$\mathrm{d}C(u, v) = \psi\left(\Psi_x^{-1}(u), \Psi_y^{-1}(v)\right) \left|\frac{\partial(u, v)}{\partial(x, y)}\right| \mathrm{d}u\mathrm{d}v$$

but should read

$$\mathrm{d}C(u, v) = \psi\left(\Psi_x^{-1}(u), \Psi_y^{-1}(v)\right) \left|\frac{\partial(u, v)}{\partial(x, y)}\right|^{-1} \mathrm{d}u\mathrm{d}v$$

v. Chapter 5, section 5.2, end of unnumbered footnote at bottom of page 45 reads

```
/d { rand 2147483647. div } det /r { d d d add add } def /normal { r r r r add add add 6 sub } def
/T 415 def /dt 1 def /sigma .5 def /a -.25 def
0 0 dt T { pop dup a mul dt mul sigma dt sqrt mul normal mul add dup dt exch rlineto add } for
```

but should read

```
/d { rand 2147483647. div } def /r { d d d add add } def /normal { r r r r add add add 6 sub } def
/T 415 def /dt 1 def /sigma .5 def /a -.25 def
0 0 dt T { pop dup a mul dt mul sigma dt sqrt mul normal mul add dup dt exch rlineto add } for
```

vi. Chapter 5, section 5.2.1, page 48, the expression between equations (5.22) and (5.23) reads

$$\eta \in [-1, 1] \quad \text{for} \quad \rho \in [-1, 1] \ .$$

but should read

$$\eta(\rho) = 2 \cdot \frac{3}{\pi} \cdot \arcsin\left(\frac{\rho}{2}\right) \quad \text{with} \quad \eta \in [-1, 1] \quad \text{for} \quad \rho \in [-1, 1] \ .$$

vii. Chapter 5, section 5.2.3, page 57, equation (5.45) reads

$$\tau_{\text{Ali-Mikhail-Haq}} = \left(\frac{3\theta - 2}{\theta}\right) - \frac{2}{3}\left(1 - \frac{1}{\theta}\right)^2 \cdot \ln(1 - \theta) \ . \tag{5.45}$$

but should read

$$\tau_{\text{Ali-Mikhail-Haq}} = \left(\frac{3\theta - 2}{3\theta}\right) - \frac{2}{3}\left(1 - \frac{1}{\theta}\right)^2 \cdot \ln(1 - \theta) \ . \tag{5.45}$$

The difference is a divisor of 3 in the first term of Kendall's τ for the Ali-Mikhail-Haq copula.

viii. Chapter 8, section 8.3.1, the section including equation (8.13), page 81 reads

A polynomial $P(z)$ of degree g,

$$P(z) = \sum_{j=0}^{g} a_k z^{g-j} \ , \tag{8.13}$$

is considered to be an element of the ring $GF[m, z]$ of polynomials over the finite field $GF[m]$ if we assume all of the coefficients a_k to be $\in GF[m]$. In other words, all algebra on the coefficients a_k is to be carried out modulo m. ...

but should read

A polynomial $P(z)$ of degree g,

$$P(z) = \sum_{j=0}^{g} a_j z^{g-j} \ , \tag{8.13}$$

is considered to be an element of the ring $GF[m, z]$ of polynomials over the finite field $GF[m]$ if we assume all of the coefficients a_j to be $\in GF[m]$. In other words, all algebra on the coefficients a_j is to be carried out modulo m. ...

The difference is that the index of the coefficients $a_.$ should be j instead of k in three instances.

ix. Chapter 8, section 8.3.2, equation (8.20), page 83 reads

$$x_{nk} := \sum_{j=1}^{d} {}^{\oplus_2} v_{kj} \mathbf{1}_{\{j^{\text{th}} \text{ bit (counting from the right) of } \gamma(n) \text{ is set}\}} \quad \text{with} \quad k = 1..d \ .$$

but should read

$$x_{nk} := \sum_{j=1}^{b} {}^{\oplus_2} v_{kj} \mathbf{1}_{\{j^{\text{th}} \text{ bit (counting from the right) of } \gamma(n) \text{ is set}\}} \quad \text{with} \quad k = 1..d \ .$$

The difference is that the sum should terminate at b, *not* d.

x. Chapter 8, section 8.3.3, equation (8.23), page 84 reads

$$x_{nk} = x_{(n-1)k} \oplus_2 v_{jk} \ . \tag{8.23}$$

but should read

$$x_{nk} = x_{(n-1)k} \oplus_2 v_{kj} \ . \tag{8.23}$$

xi. Chapter 8, section 8.3.4, page 86, middle of first paragraph reads

.... The original article [Sob76], for instance, provides initialisation numbers up to dimension 16 for property A and up to dimension 6 for property A', and some authors have gone through some considerable effort for higher dimensions, namely Paskov and Traub calculated them up until dimension 360 [PT95]. In finance, however, we often face problems that are of very high dimensionality d. ...

but should read

.... The original article [Sob76], for instance, provides initialisation numbers up to dimension 16 for property A and up to dimension 6 for property A'. Unfortunately, the importance of these properties has not been fully appreciated by many subsequent publications on the subject of usability of Sobol' numbers in higher dimensions, which may be the reason that many authors resort to various forms of enhancements [Owe98] already in very moderate dimensionalities. In finance, however, we often face problems that are of very high dimensionality d. ...

I was informed by readers that Paskov and Traub's methods don't actually have the properties I thought they have. This was a mistake in my research (I misinterpreted some statements in an article by Paskov and Traub).

xii. Chapter 8, section 8.3.4, page 86, equation (8.25) reads

$$w_{kl} := \text{int} \left[u_{kl}^* \cdot 2^{l-1} \right] \qquad \text{but should read} \qquad w_{kl} := \text{int} \left[u_{kl}^* \cdot 2^l \right] \ .$$

xiii. Chapter 10, section 10.2, equation (10.3), page 112 reads

$$\frac{\partial v}{\partial p} \approx \frac{v(p + \Delta p) - v(p)}{\delta p} \ . \qquad \text{but should read} \qquad \frac{\partial v}{\partial p} \approx \frac{v(p + \Delta p) - v(p)}{\Delta p} \ .$$

The difference is that the denominator on the right hand side should start with an upper case Δ, *not* a lower case δ.

xiv. Chapter 10, section 10.2, equations (10.4) to (10.5), page 112 and 113 read

$$
\begin{aligned}
\mathrm{V}\left[\frac{\partial v}{\partial p}\right] &\approx \frac{1}{\Delta p}\cdot\ldots \\
&= \frac{1}{\Delta p}\cdot\ldots \\
&= \frac{1}{\Delta p}\cdot\ldots \\
&= \frac{1}{\Delta p}\cdot\ldots \\
&\approx \frac{2}{\Delta p}\cdot\ldots
\end{aligned}
\qquad\text{and should read}\qquad
\begin{aligned}
\mathrm{V}\left[\frac{\partial v}{\partial p}\right] &\approx \frac{1}{(\Delta p)^2}\cdot\ldots \\
&= \frac{1}{(\Delta p)^2}\cdot\ldots \\
&= \frac{1}{(\Delta p)^2}\cdot\ldots \\
&= \frac{1}{(\Delta p)^2}\cdot\ldots \\
&\approx \frac{2}{(\Delta p)^2}\cdot\ldots
\end{aligned}
$$

The denominator of the multiplicative quotient at the beginning of the right hand side should always be $(\Delta p)^2$, *not* simply Δp.

xv. Chapter 10, section 10.8.3, equation (10.36), page 126 reads

$$
\mathrm{E}\left[B_{t_j}\right] = \left(\frac{v_k - v_j}{v_k - v_j}\right)B_{t_i} + \left(\frac{v_j - v_i}{v_k - v_i}\right)B_{t_k}
$$

and should read

$$
\mathrm{E}\left[B_{t_j}\right] = \left(\frac{v_k - v_j}{v_k - v_i}\right)B_{t_i} + \left(\frac{v_j - v_i}{v_k - v_i}\right)B_{t_k}
$$

xvi. Chapter 10, section 10.8.3, bottom of page 126, replace

```
leftWeight[i]  = (v[k]-v[1])/(v[k]-v[j-1]);
rightWeight[i] = (v[1]-v[j-1])/(v[k]-v[j-1]);
stddev[i]      = sqrt((v[1]-v[j-1])*(v[k]-v[1])/v[k]-v[j-1]));
```

with

```
const double vjm1 = j ? v[j-1] : 0., dv = v[k]-vjm1;
leftWeight[i]    = (v[k]-v[l]) / dv;
rightWeight[i]   = (v[l]-vjm1) / dv;
stddev[i]        = sqrt( (v[l]-vjm1) * (v[k]-v[l]) / dv );
```

xvii. Chapter 10, section 10.8, top of page 127 reads

```
vector <unsigned long> leftIndex, rightIndex, bridgeIndex;
vector <double> leftWeight, rightWeight, sigma;
```

and should read

```
std::vector <unsigned long> leftIndex, rightIndex, bridgeIndex;
std::vector <double> leftWeight, rightWeight, stddev;
```

xviii. Chapter 10, section 10.9.1, equation (10.51), page 135 reads

$$
D_n(\lambda) = (1 - 2\lambda)D_{n-1}(\lambda) - \lambda
\begin{vmatrix}
\lambda & 1 & 1 & \cdots & 0 \\
\lambda & 2-\lambda & 2 & \cdots & 2 \\
\lambda & 2 & 3-\lambda & \cdots & 3 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\lambda & 2 & 3 & \cdots & n-1-\lambda
\end{vmatrix}.
$$

and should read

$$D_n(\lambda) = (1 - 2\lambda)D_{n-1}(\lambda) - \lambda \begin{vmatrix} \lambda & 1 & 1 & \cdots & 1 \\ \lambda & 2-\lambda & 2 & \cdots & 2 \\ \lambda & 2 & 3-\lambda & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda & 2 & 3 & \cdots & n-1-\lambda \end{vmatrix} .$$

xix. Chapter 10, section 10.9.1, equation (10.59), page 136 reads

$$D_n(\lambda) = \left(\frac{1 - \lambda - \alpha_-}{\alpha_+ - \alpha_-} \right) \alpha_+^n - \left(\frac{1 - \lambda - \alpha_+}{\alpha_+ - \alpha_-} \right) \alpha_-^{-n}$$

and should read

$$D_n(\lambda) = \left(\frac{1 - \lambda - \alpha_-}{\alpha_+ - \alpha_-} \right) \alpha_+^n - \left(\frac{1 - \lambda - \alpha_+}{\alpha_+ - \alpha_-} \right) \alpha_-^n$$

xx. Chapter 10, section 10.9.1, equation (10.68), page 136 reads

$$C_n = S_n \cdot \Lambda_n \cdot S_N{}^\top$$

and should read

$$C_n = S_n \cdot \Lambda_n \cdot S_n{}^\top$$

xxi. Chapter 10, section 10.9.2, equation (10.76), page 137 reads

$$W_{t_k} = W_{t_j} + \sqrt{t_k - t_i}\, z$$

and should read

$$W_{t_k} = W_{t_i} + \sqrt{t_k - t_i}\, z$$

xxii. Chapter 10, section 10.9.2, equation (10.78), page 138 reads

$$y(x, z) = \frac{\sqrt{t_k - t_i}\, z - \sqrt{t_j - t_i}\, x}{\sqrt{t_k - t_i}}$$

and should read

$$y(x, z) = \frac{\sqrt{t_k - t_i}\, z - \sqrt{t_j - t_i}\, x}{\sqrt{t_k - t_j}}$$

xxiii. Chapter 10, section 10.9.2, equation (10.79), page 138 reads

$$= \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2} \left(\frac{x - \sqrt{\frac{t_j - t_i}{t_k - t_i}}\, z}{\frac{t_k - t_j}{t_k - t_i}} \right)^2}$$

and should read

$$= \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2} \left(\frac{x - \sqrt{\frac{t_j - t_i}{t_k - t_i}}\, z}{\sqrt{\frac{t_k - t_j}{t_k - t_i}}} \right)^2}$$

xxiv. Chapter 12, section 12.4, equation (12.20), page 166 reads

$$\frac{\partial P_{i+1}}{\partial f_k} = -P_{i+1} \frac{\tau_k}{1 + f_k \tau_k} \cdot \mathbf{1}_{\{k \geq i\}},$$

and should read

$$\frac{\partial P_{i+1}}{\partial f_k} = -P_{i+1} \frac{\tau_k}{1 + f_k \tau_k} \cdot \mathbf{1}_{\{k \leq i\}} \quad \text{for} \quad i, k < n,$$

xxv. Chapter 12, section 12.4, equation (12.24), page 167 reads

$$\hat{\sigma}_{SR_i}(t, T) = \sqrt{\sum_{k=i,l=i}^{n-1} Z_{ik}^{FRA \to SR}(0) \cdot \frac{\int_t^T \sigma_k(t')\sigma_l(t')\rho_{kl}\mathrm{d}t'}{T} \cdot Z_{il}^{FRA \to SR}(0)}$$

and should read

$$\hat{\sigma}_{SR_i}(t, T) = \sqrt{\sum_{k=i,l=i}^{n-1} Z_{ik}^{FRA \to SR}(0) \cdot \frac{\int_t^T \sigma_k(t')\sigma_l(t')\rho_{kl}\mathrm{d}t'}{T - t} \cdot Z_{il}^{FRA \to SR}(0)}$$

xxvi. Chapter 13, section 13.3, equation (13.24), page 190 reads

$$S^{(4)} = \begin{pmatrix} -1.1588 & -0.91287 & -0.6455 & -0.5 \\ 1.1588 & -0.91287 & -0.6455 & -0.5 \\ 0 & 1.8257 & -0.6455 & -0.5 \\ 0 & 0 & 1.9365 & -0.5 \\ 0 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{R^{(m)}} S'^{(4)} = \begin{pmatrix} -1.1306 & -1.1053 & 0 & 1.22474 \\ 1.1053 & -1.1306 & 0.91287 & -0.8165 \\ 0 & 0 & -1.8257 & -0.8165 \\ -1.1053 & 1.1306 & 0.91287 & -0.8165 \\ 1.1306 & 1.1053 & 0 & 1.22474 \end{pmatrix}.$$

and should read

$$S^{(4)} = \begin{pmatrix} -1.5811 & -0.91287 & -0.6455 & -0.5 \\ 1.5811 & -0.91287 & -0.6455 & -0.5 \\ 0 & 1.8257 & -0.6455 & -0.5 \\ 0 & 0 & 1.9365 & -0.5 \\ 0 & 0 & 0 & 2 \end{pmatrix} \xrightarrow{R^{(m)}} S'^{(4)} = \begin{pmatrix} -1.1306 & -1.1053 & 0 & 1.22474 \\ 1.1053 & -1.1306 & 0.91287 & -0.8165 \\ 0 & 0 & -1.8257 & -0.8165 \\ -1.1053 & 1.1306 & 0.91287 & -0.8165 \\ 1.1306 & 1.1053 & 0 & 1.22474 \end{pmatrix}.$$

The difference is in the two top entries in the leftmost column of the matrix on the left hand side. They should read -1.5811 and 1.5811, *not* -1.1588 and 1.1588.

xxvii. Chapter 13, section 13.5, table 13.1, page 192 reads

$\cdots$	discount factor	f_i	$\cdots$
$\vdots$	0.762757096	6.652%	$\vdots$
$\vdots$	0.739640975	6.251%	$\vdots$
$\vdots$	0.717225412	6.044%	$\vdots$
$\vdots$	0.696187117	6.044%	$\vdots$
$\vdots$	0.675765937		$\vdots$

and should read

$\cdots$	discount factor	f_i	$\cdots$
$\vdots$	0.762757096	6.200%	$\vdots$
$\vdots$	0.739640975	6.251%	$\vdots$
$\vdots$	0.717225412	6.044%	$\vdots$
$\vdots$	0.696187117	6.044%	$\vdots$
$\vdots$	0.675765937		$\vdots$

The difference is the forward rate entry in the first row. This should not have caused much of a problem since it is easy to compute the forward rates from the given discount factors:

$$\left(\frac{0.762757096}{0.739640975} - 1\right) \Big/ (4.5 - 4) \approx 6.200\%$$

The references listed below are already correctly covered in the book's bibliography, thus no changes to the bibliography are needed.

References

[MWCM98] D. J. Medeiros, E. F. Watson, J. S. Carson, and M. S. Manivannan, editors. *Proceedings of the 1998 Winter Simulation Conference*, 1998.

[Owe98] A.B. Owen. Monte Carlo extension of quasi-Monte Carlo. In Medeiros et al. [MWCM98]. www.informs-cs.org/wsc98papers/076.PDF.

[PT95] S. H. Paskov and J. F. Traub. Faster Valuation of Financial Derivatives. *Journal of Portfolio Management*, 22(1):113–120, 1995. There are also several Columbia University technical reports on the subject.

[Sob76] I. M. Sobol'. Uniformly Distributed Sequences with an Additional Uniform Property. *USSR Computational Mathematics and Mathematical Physics*, 16(5):236–242, 1976.